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Dynamic Games among Teams with Asymmetric Information

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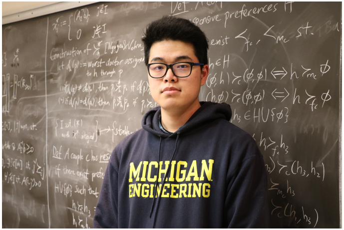
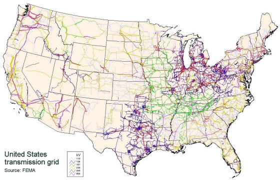
Collaborators

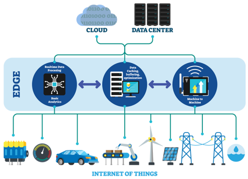
- **Dengwang Tang** (Ebay), Hamidreza Tavafoghi (Google), Ashutosh Nayyar (USC) & Demosthenis Teneketzis (UMichigan)



Multi-Agent (Dynamic) Systems

- Multiple agents take actions *over time* on top of an *ever changing* system/environment to achieve their respective objectives.
- Agents have different information about the systems and about each other.
- Multi-agent systems have a wide range of applications.





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Multi-Agent (Dynamic) Systems

- *Team*: Agents have aligned objectives and jointly decide on their strategies.
- *Game*: Agents have different objectives and do not collaborate.
- **Focus of this talk**: *Games among teams*

Motivation

- Automated vehicles in future cities
  - Automated vehicles roam around the city to serve customers.
  - Vehicles from the same company collaborate to serve customers more efficiently.
  - Fleets from different companies compete with each other.

- DARPA Spectrum Collaboration Challenge
  - Transceivers work in teams.
  - Teams compete with each other for spectrum resource.
  - A team collaborates to sense the spectrum landscape, avoid interference, and exploit opportunities to achieve efficient use of spectrum.

Information Asymmetry

- Information asymmetry is ubiquitous.
- It exists among all agents, whether they are in the same team or not.
- Why cannot members in the same team simply communicate to overcome information asymmetry?
  - Physical constraints
  - Communication delay vs. real-time requirement
  - High cost of communication
  - Possibility of being eavesdropped by other teams

Challenges

- Challenges of dynamic games with asymmetric information, among individuals or among Teams:
  - Interdependence of decision and information over time
  - Domain of strategy growing over time
  - *Dependence of belief on strategies*
  - Coordination within teams to achieve team optimality
- **This talk**: Show how to address these challenges in a specialized model [1].

[1] D. Tang, H. Tavafoghi, V. S., A. Nayyar and D. Teneketzis (2023). Dynamic games among teams with delayed intra-team information sharing. *Dynamic Games and Applications*, 13(1), 353-411.

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“Existing Work” on Dynamic Games

- Repeated games.
- Zero-sum dynamic games.
- [Maskin, Tirole 2001]
  - *Model assumption*: The system states and actions are publicly observable.
  - *Result*: Agents can make decisions based on the Markov state instead of full information. A sequential decomposition procedure to determine such Perfect equilibria.
- [Nayyar, Gupta, Langbort, Basar 2013]
  - *Model assumption*: Common information based belief is strategy independent.
  - *Result*: Agents can make decisions based on a compression of common information. A sequential decomposition procedure to determine such Bayes Nash equilibria.
- [Ouyang, Tavafoghi, Teneketzis 2016]
  - Similar model to ours, except in games of individuals.
  - *Result*: A sequential decomposition procedure to determine compression based Weak Perfect Bayesian equilibria.

"Existing Work" on Dynamic Games

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- [Vasal, Sinha, Anastasopoulos 2019]
  - Similar model to ours, except in games of individuals.
  - *Result*: A sequential decomposition procedure to determine compression based Weak Perfect Bayesian equilibria.
- [Ouyang, Tavafoghi, Teneketzis 2024]
  - *Model assumption*: General finite game model with *hidden actions*.
  - *Result*: Agents can make decisions based on an elaborate compression of common information that accounts for hidden actions. A sequential decomposition procedure to determine such Bayes Nash equilibria.
- [Tang, S., Teneketzis 2024] *(Under review)*
  - General finite game model.
  - *Result*: Develop criteria for strategy-independent information compression maps to preserve some or all equilibria—Bayes Nash (BN) and Sequential equilibrium.

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System Model

- We consider a **FINITE** stochastic dynamic game:
  - Time indices: $\mathcal{T} = \{1, 2, \cdots, T\}$, $T < +\infty$.
  - Set of teams: $\mathcal{I} = \{1, 2, \cdots, I\}$, $I < +\infty$.
  - Set of Team i's members: $\mathcal{N}_i = \{(i,1), (i,2), \cdots, (i, N_i)\}$, $N_i < +\infty$.

- At time $t \in \mathcal{T}$:
  - Agent (i,j)'s action (finite-valued): $U_t^{i,j}$.
  - Team i's dynamical system (finite-valued): $\mathbf{X}_t^i = (X_t^{i,j})_{(i,j)\in\mathcal{N}_i}$.
  - System evolution:

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t, W_t^{i,X}), \quad i \in \mathcal{I}$$

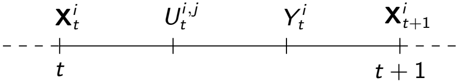
where $\mathbf{U}_t = (U_t^{k,j})_{(k,j)\in\mathcal{N}_k, k\in\mathcal{I}}$ is the action profile, and $(W_t^{i,X})_{i\in\mathcal{I}, t\in\mathcal{T}}$ are the system noises.

Information Structure

- All actions are publicly observable.
- Noisy public observation: $\mathbf{Y}_t = (Y_t^i)_{i \in \mathcal{I}}$

$$Y_t^i = \ell_t^i(\mathbf{X}_t^i, \mathbf{U}_t, W_t^{i,Y})$$

  where $(W_t^{i,Y})_{i \in \mathcal{I}, t \in \mathcal{T}}$ are the observation noises.
- Timeline:



- Agent (i,j) can observe $X_t^{i,j}$ perfectly.
- Agents in the same team communicate their states with a delay of d.
- $(\mathbf{X}_1^i)_{i \in \mathcal{I}}, (W_t^{i,X})_{i \in \mathcal{I}, t \in \mathcal{T}}, (W_t^{i,Y})_{i \in \mathcal{I}, t \in \mathcal{T}}$ are mutually independent primitive random variables.
- Model specification is common knowledge among all agents.

Information Structure

- Common information among all teams:

$$H_t^0 = (\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1})$$

- Common information within team i:

$$H_t^i = (\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1}, \mathbf{X}^i_{1:t-d})$$

- Agent (i, j)'s information:

$$H_t^{i,j} = (\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1}, \mathbf{X}^i_{1:t-d}, X^{i,j}_{t-d+1:t})$$

Reward Functions

- Team i's strategy profile:
  - Pure strategy: $\mu^i = (\mu_t^{i,j})_{(i,j)\in\mathcal{N}_i, t\in\mathcal{T}}$ where $\mu_t^{i,j} : \mathcal{H}_t^{i,j} \mapsto \mathcal{U}_t^{i,j}$.
  - Mixed strategies: σ^i, a distribution over pure strategies.
    - Team members' strategies can be correlated!
    - Joint randomization is critical for equilibrium existence!
- Total reward
  - Total reward under a pure strategy profile $\mu = (\mu^i)_{i\in\mathcal{I}}$:

$$J^i(\mu) = \mathbb{E}^{\mu}\left[\sum_{t=1}^{T} r_t^i(\mathbf{X}_t, \mathbf{U}_t)\right],$$

  where the functions r_t^i represents the instantaneous rewards.
  - Total reward under a mixed strategy profile: average of total rewards under pure strategy profiles, i.e.

$$J^i(\sigma) = \sum_{\mu} J^i(\mu) \prod_{i\in\mathcal{I}} \sigma^i(\mu^i)$$

Solution Concept

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- Team Bayesian Nash Equilibrium (TBNE): A mixed strategy profile $\sigma = (\sigma^i)_{i \in \mathcal{I}}$ where
$$J^i(\sigma^i, \sigma^{-i}) \geq J^i(\tilde{\sigma}^i, \sigma^{-i})$$
for all mixed strategy profile $\tilde{\sigma}^i$ of team i.

- **Objective:** *While preserving the given information structure*
  - Characterize sub-classes of TBNE where agents can compress their information
  - Devise a sequential decomposition procedure to determine these TBNE

Deviations can be arbitrarily complex (given the information structure)!

Result Overview

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- We transform the game among teams into a game among individual players, where the players are *coordinators*.
  - The game among coordinators will be a dynamic game with asymmetric information.
    Coordinators will have *common information* and *private information*.
- We introduce a subclass of strategies where *coordinators* compress their *private information*.
  - We show that equilibria always exist in this subclass.
- We introduce a subclass of strategies where *coordinators* compress both their *common information* and *private information*.
  - We develop a sequential decomposition procedure whose solution (if it exists) forms an equilibrium in this subclass.
  - We show that equilibria may not exist in this subclass.

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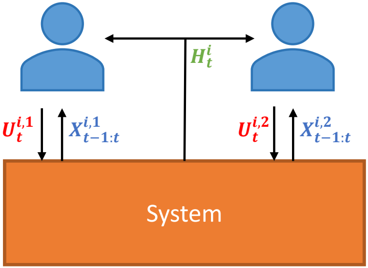
Coordinators' Approach [NMT 2013]



Coordinators' Approach [NMT 2013]

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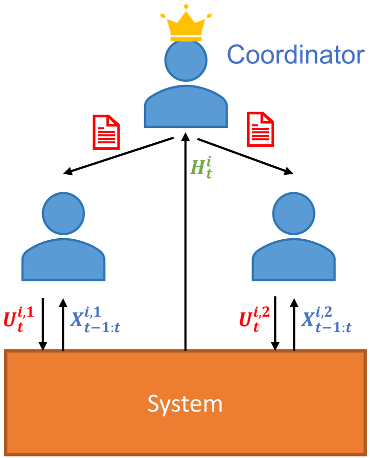

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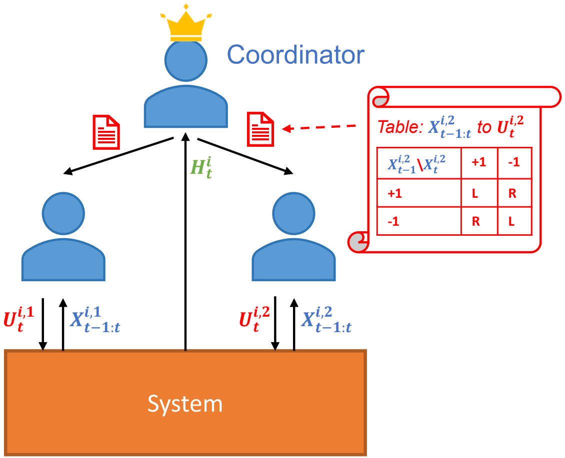

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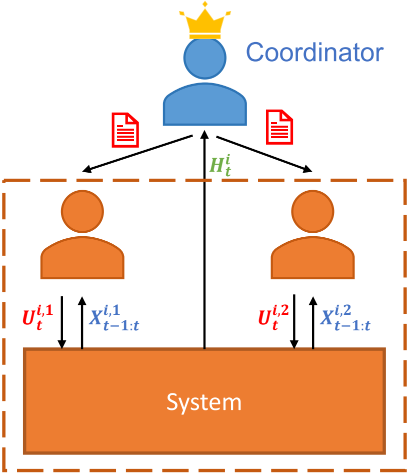

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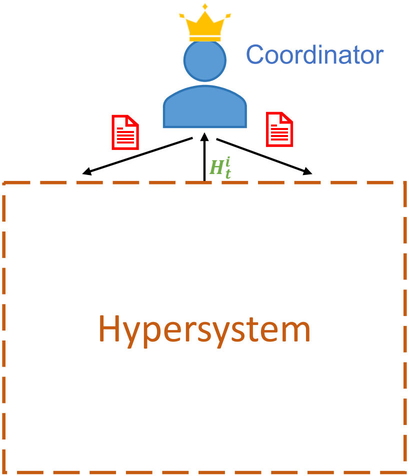

Game among Coordinators

- The transformation neither enriches nor reduces the strategy space of a team—joint randomization within team is critical for this!
- Coordinator i's "action" at time t: $\mathbf{\Gamma}_t^i = (\Gamma_t^{i,j})_{(i,j)\in\mathcal{N}_i}$.
  - $\Gamma_t^{i,j}$ instructs agent (i,j) to choose her action.
- Behavioral strategies of a coordinator (Behavioral Coordination Strategies): $g_t^i : \mathcal{H}_t^i \times \mathit{\Gamma}_{1:t-1}^i \mapsto \Delta(\mathit{\Gamma}_t^i)$
  - That is, choose randomized prescriptions based on H_t^i, and past prescriptions.
- Can analyze *Coordinator's Bayesian Nash Equilibrium* (CBNE) in *behavioral coordination strategies* instead! *(Kuhn's theorem applies)*

Information Structure of the Coordinators

- The game among coordinators is a dynamic game with asymmetric information.
  - All coordinators know
    $$H_t^0 = (\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1})$$
  - Coordinator i knows
    $$\overline{H}_t^i = (\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1}, \mathbf{X}_{1:t-d}^i, \mathbf{\Gamma}_{1:t-1}^i)$$
  - No coordinator knows $\mathbf{X}_{t-d+1:t}$!
  - We will refer to
    - $\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1}$ as the Common Information;
    - $\mathbf{X}_{1:t-d}^i, \mathbf{\Gamma}_{1:t-1}^i$ as coordinator i's Private Information.
    - $\mathbf{X}_{t-d+1:t}$ as the Hidden States.

Compression of Information

- Now we have completed Step 0: transform teams into coordinators.

- The game among coordinators is still complicated.
  - Both the common information and the private information are growing with time.
  - Coordinators' actions and information are interdependent over time.
  - **Coordinators' actions (prescriptions) are private information.**

- The results of [OTT 2016] do not directly apply! *(OTT 2024 does.)*

- **Key Questions:**
  - Do there exist equilibria where coordinators can compress their information?
  - Can we develop a sequential decomposition procedure to find such equilibria, if they exist?

Deviations can be arbitrarily complex!

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Compression of Private Information

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- Coordinator i's private information at time t is $(\mathbf{X}^i_{1:t-d}, \mathbf{\Gamma}^i_{1:t-1})$
- However, we show that coordinator i only need to use the *Sufficient Private Information* (SPI) S^i_t, given by

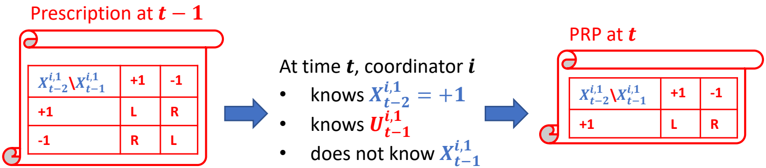
$$S^i_t = (\mathbf{X}^i_{t-d}, \mathbf{\Phi}^i_t) \tag{1}$$

where $\mathbf{\Phi}^i_t$ is the *partially realized prescriptions* (PRP).

  - PRP: the $(d-1)$ most recent prescriptions evaluated at the current information of the coordinator.

Partially Realized Prescriptions

- PRP summarizes how hidden states were mapped to actions in the past, provided what is known.
- $d = 1$: $\mathbf{\Phi}_t^i$ is empty.
- $d = 2$: $\mathbf{\Phi}_t^i = (\Gamma_{t-1}^{i,j}(X_{t-2}^{i,j}, \cdot))_{(i,j)\in\mathcal{N}_i}$.



Compression of Private Information

- We refer to such strategies as *Sufficient Private Information-Based* (SPIB) strategies, ...
  i.e. strategies that uses (H_t^0, S_t^i) instead of $(H_t^0, \mathbf{X}_{1:t-d+1}^i, \mathbf{\Gamma}_{1:t-1}^i)$ to choose randomized prescriptions.
- SPIB strategies are simpler to implement than generic behavioral coordination strategies:
  - Size of $S_t^i = (\mathbf{X}_{t-d}^i, \mathbf{\Phi}_t^i)$ does not grow with time.
  - $S_t^i = (\mathbf{X}_{t-d}^i, \mathbf{\Phi}_t^i)$ can be sequentially updated as new information comes in.

Main Result

Theorem (T., T., S., N., T.)

There exists CBNE where all coordinators use SPIB strategies.

Remark: Existence of equilibria is typically established under the assumption of full recall, but using an SPIB strategy means giving up full recall.

Proof Sketch:

- Restrict attention to ϵ-trembling strategies of coordinators, where all prescriptions have probability $\geq \epsilon$.
- Fixing g^{-i}, coordinator i's decision problem is equivalent to an MDP with state (H_t^0, S_t^i).
- Define a best response correspondence using the sequential decomposition of this MDP.
- Using Kakutani's fixed point theorem to argue the existence of ϵ-trembling-SPIB-strategy-based equilibria.
- Take the limit as ϵ goes to 0.

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Compression of Common Information

- Recall: Coordinator i's information is $(\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1}, \mathbf{X}^i_{1:t-d}, \mathbf{\Gamma}^i_{1:t-1})$
  - We have compressed the private information $(\mathbf{X}^i_{1:t-d}, \mathbf{\Gamma}^i_{1:t-1})$ to S^i_t which has a time invariant domain.
  - The common information $(\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1})$ is still growing with time!
- How can we compress the common information?

  And how can we develop a sequential decomposition of the game?

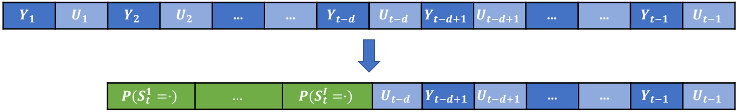
  Motivation:
  - In decentralized control [NMT 2013] ...
    and multiple settings of dynamic games
    [NGLB 2013][OTT 2016][VSA 2019],...
    common information based belief on private information and state forms an information state.
  - Sequential decomposition procedures are developed based on this information state.

Compression of Common Information

- Idea: Compress the common information to the belief on sufficient private information and hidden states, i.e.

$$\mathbb{P}(\mathbf{S}_t = \cdot, \mathbf{X}_{t-d+1:t} = \cdot | H_t^0)$$

- Using special properties of this model, it can be shown that this belief can be expressed as a function of
  - Beliefs on SPI, i.e. $\mathbb{P}(S_t^k = \cdot | H_t^0)$ for each $k \in \mathcal{I}$.
  - Uncompressed values of $(\mathbf{Y}_{t-d+1:t-1}, \mathbf{U}_{t-d:t-1})$.
- Refined Idea: Compress the common information to B_t, a vector composed of the above objects.



Compression of Common Information

- However, the beliefs $\mathbb{P}(S_t^k = \cdot | H_t^0), k \in \mathcal{I}$ are strategy dependent.
  - It can be sequentially computed with Bayes rule at each time t, but the rule depends on the strategy profile of the coordinators.
- Unlike the compression of private information into S_t^i, there is no fixed way to compute the compression of common information.
- We have to determine the compression and the strategies at the same time.

Compressed Information Based Strategy

- A *Compressed Information-Based* (CIB) strategy profile is composed of two parts:
  - *Belief generation system* ψ^i:
    Defines how coordinator i *sequentially updates* the beliefs $\Pi_t^{i,k} = \mathbb{P}(S_t^k = \cdot | H_t^0), k \in \mathcal{I}$ from H_t^0.
  - *Belief based strategy* λ^i:
    Defines how coordinator i map all her compressed information to prescriptions.

$$
\left.\begin{array}{l}
\left.\begin{array}{r}
(\mathbf{Y}_{1:t-1}, \mathbf{U}_{1:t-1}) \xrightarrow{\psi^i_{1:t-1}} \Pi_t^{i,1:I} \\
(\mathbf{Y}_{t-d+1:t-1}, \mathbf{U}_{t-d:t-1})
\end{array}\right\} B_t^i \\
(\mathbf{X}_{1:t-d}, \mathbf{\Gamma}_{1:t-d}) \longrightarrow S_t^i
\end{array}\right\} \xrightarrow{\lambda_t^i} \text{(randomized) } \Gamma_t^i
$$

- CIB strategies forms a subclass of SPIB strategies.

Compressed Information Based Strategy

- We are interested in CIB strategies with reasonable belief generation systems.
- Following similar approach in [OTT 2016], we focus on the subclass of CBNE (called CIB-CBNE) where
  i. Coordinator i plays CIB strategy profile (λ^i, ψ^i)
  ii. $\psi^i = \psi^*$ for all $i \in \mathcal{I}$;
  iii. ψ^i is consistent with $(\lambda^k)_{k\in\mathcal{I}}$ under Bayes rule for all $i \in \mathcal{I}$;
- When condition (ii) holds, all coordinators have the same *compressed common information* B_t, and it is common knowledge.
- When condition (iii) holds, the common information-based beliefs generated by the coordinators are objective beliefs.

Sequential Decomposition

- When the condition of CIB-CBNE holds, the *compressed common information* B_t can be seen as a summary of the past that is agreed by all coordinators.
    - It can be use to derive objective beliefs on the states and information at or after time t.

- The decision problem for the coordinators at time t can be seen as a new game (starting at time 0) parametrized by B_t (Call it $G_{\geq t}(B_t)$.)

  Same observation holds true for $t+1$!

- This observation suggests a sequentially decomposition procedure where the game is decomposed into *stage problems* with information state B_t.

Stage Problem

- Suppose that we have already know the interim expected payoff-to-go at equilibrium in $G_{\geq t+1}(b_{t+1})$ for every realization of CCI b_{t+1}, given by the value functions $V_{t+1} = (V^i_{t+1})_{i\in\mathcal{I}}$.
- Suppose that we restrict all coordinators to use belief update rule ψ^*_t, then coordinators are essentially playing the following game at time t:

  Stage Game $G_t(V_{t+1}, b_t, \psi^*_t)$:

  - Value functions V_{t+1}, realization of CCI b_t, belief update rule ψ^*_t are commonly known.
  - Coordinator i observes her type $S^i_t = s^i_t$.
  - Coordinator i selects a prescription $\mathbf{\Gamma}^i_t$.
  - Coordinator i has utility

  $$\mathbb{E}[r^i_t(\mathbf{X}_t, \mathbf{U}_t) + V^i_{t+1}(B_{t+1}, S^i_{t+1})|b_t, s^i_t, \mathbf{\Gamma}^i_t]$$

  where the beliefs in B_{t+1} is computed with ψ^*_t and b_t.

- If we can find an equilibrium strategy profile of the above game that is consistent with ψ^*_t under Bayes rule, we have found an equilibrium for $G_{\geq t}(b_t)$. (Why?)

Closedness of CIB Strategies

- This is because of the *closedness property*:

Lemma (T., T., S., N., T.)

Suppose that all coordinators other than coordinator i are using CIB strategies. Let $(\lambda^k, \psi^k)_{k\in\mathcal{I}\setminus\{i\}}$ be the CIB strategy profile of coordinators other than i. Suppose that

- $\psi^j = \psi^k$ *for all* $j, k \in \mathcal{I}\setminus\{i\}$
- ψ^j *is consistent with* $(\lambda^k)_{k\in\mathcal{I}\setminus\{i\}}$ *under Bayes rule.*

Then, a best-response strategy for coordinator i is a CIB strategy with the same belief generation system as the other coordinators.

- *Proof Idea*: Fixing all other coordinators' strategies, let B_t^i be sequentially computed with $\psi^i = \psi^k, k \neq i$. B_t^i contains the true belief of other coordinators' SPI. Then one can argue that coordinator i faces an MDP with state (B_t^i, S_t^i).

Sequential Decomposition

Theorem (T., T., S., N., T.)

Let $(\lambda^{*i}, \psi^*)_{i \in \mathcal{I}}$ be a CIB strategy profile with identical belief generation system ψ^* for all $i \in \mathcal{I}$. If this strategy profile satisfies the dynamic program:

$$V^i_{T+1}(\cdot,\cdot) = 0 \quad \forall i \in \mathcal{I};$$

and for $t \in \mathcal{T}, b_t \in \mathcal{B}_t$

- $\lambda^*_t(b_t,\cdot)$ *is an Interim BNE of the stage game* $G_t(V_{t+1}, b_t, \psi^*_t)$*;*
- $\psi^*_t(b_t,\cdot)$ *is consistent with* $\lambda^*_t(b_t,\cdot)$ *under Bayes rule;*
- $V^i_t(b_t, s^i_t)$ *is the interim expected utility of player i in the stage game* $G_t(V_{t+1}, b_t, \psi^*_t)$ *for all* $s^i_t \in \mathcal{S}^i_t$ *and* $i \in \mathcal{I}$*,*

then $(\lambda^{*i}, \psi^*)_{i \in \mathcal{I}}$ forms an CBNE.

- The sequential decomposition procedure does not always have a solution—the paper provides two instances where a solution exists.

Conclusions

- Dynamic games among teams with asymmetric information has many applications.
- We transform games among teams to games among individuals with the coordinator's approach.
- We developed a general approach to characterize a subset of Nash Equilibria with the following feature: At each time, each agent can make their decision based on a compressed version of their information.
- We identified two subclasses of strategies:
  - Sufficient private information-based (SPIB) strategies, which only compresses private information;
  - Compressed information-based (CIB) strategies, which compresses both common and private information.
- We showed that while SPIB-strategy-based equilibria always exist, CIB strategy-based equilibria do not always exist.
- We developed a backward inductive sequential procedure, whose solution (if it exists) is a CIB strategy-based equilibrium.

Gracie mille!
Thank you!

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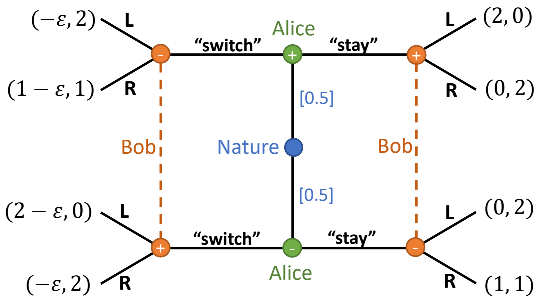
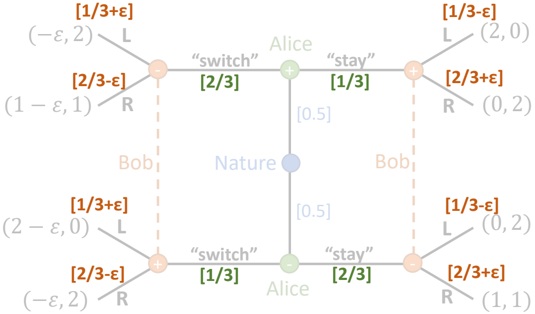
Existence I

- In fact, CIB-CBNE does not always exist!
  Example:



Existence II

- There is only one Bayes Nash Equilibrium in the above game. In this equilibrium both players play mixed strategies:

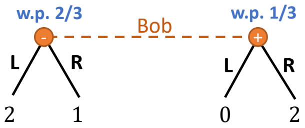


Existence

- Given Alice's strategy, the Bayes consistent belief at stage 2 satisfies:

$$\mathbb{P}(\text{“+”} \mid \text{“switch”}) = \mathbb{P}(\text{“+”} \mid \text{“stay”}) = \frac{1}{3}$$

- Bob faces the same decision problem under “switch” and “stay,” i.e.



  - Bob is indifferent between L and R in this situation.
- However, at equilibrium, Bob chooses different randomization under “switch” and “stay” ...
  - Bob uses more than his belief to make decision!